Propagation of Quasi-Static Modes in Anisotropic Transmission Lines: Application to MIC Lines

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Abstract —In this paper, we analyze the field propagation in a general N-conductor transmission line embedded in an inhomogeneous and anisotropic medium, through the series expansion of the field in powers of frequency. The quasi-static approach is deduced as a zero-order approach upon the field and a first-order approach for the propagation constant. It is shown that it is even possible to decompose the field into a sum of propagating modes with a scalar propagation factor.

The special case of transmission lines in nonmagnetic media is explicitly considered. A method to find out the mode characteristics of any open planar MIC line with anisotropic dielectric substrates is developed and applied to some MIC structures of interest, specifically broadside edge-coupled microstrips with inverted and noninverted substrates.

I. INTRODUCTION

LATELY, transmission lines embedded in inhomogeneous and anisotropic media (such as microstrips, coplanar waveguides, coupled slots, etc.) have received considerable theoretical and practical interest. These structures are commonly analyzed under a quasi-static approach. Nevertheless, there is not, at the moment, any general study of the limits and most general features of that approach when it is applied to the general multiconductor transmission line in anisotropic inhomogeneous media, at least as far as we know.

An analytical justification of the quasi-static approach for these structures, but in isotropic media, was made by A. F. dos Santos *et al.* [1] and by I. V. Lindell [2] through the series expansion in powers of frequency of the field quantities. In the present paper, we consider the more general structure mentioned above under the same point of view in order to generalize those results, if possible. The special and important case when the medium is nonmagnetic is explicitly analyzed.

The modal decomposition of propagating electromagnetic fields arises from the analysis as a consequence of the symmetry of the inductance and capacitance matrices of the line. A method to perform that decomposition is provided. When it is applied to some known MIC lines on anisotropic substrates, some interesting features appear, which are developed in the examples.

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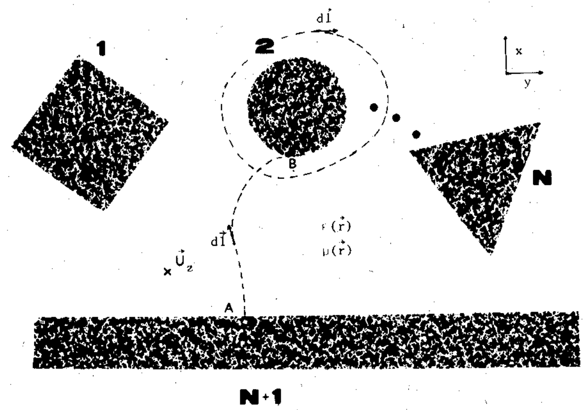

Fig. 1. Schematic cross section of a multiconductor transmission line embedded in an inhomogeneous and anisotropic medium.

II. ANALYSIS

A. Field Expansion

Let us consider the general structure in Fig. 1. It is an $N+1$ perfect conductor system, embedded in an inhomogeneous anisotropic medium, of which the dielectric permittivity tensor $\bar{\bar{\epsilon}}$ and magnetic permeability tensor $\bar{\bar{\mu}}$ are unspecified functions of frequency of the position in the transverse $x-y$ plane and, eventually, of some external parameters. The static conductivity of the medium is zero, and the structure is invariant under translations along the z-axis.

We suppose that all of the meaningful physical quantities can be developed as a power series of frequency

$$A = A_0 + A_1 \cdot \omega + A_2 \cdot \omega^2 + \cdots . \tag{1}$$

If a propagation of the kind $A = A'(x, y) \cdot e^{-j\beta z + j\omega t}$ is imposed, the even coefficients of the complex field vectors must be real numbers, and the odd ones imaginary numbers. This can be deduced from (1), using general considerations on the character of the complex field quantities [2]. The inverse statement is valid for the propagation factor: the even coefficients are imaginary and the odd coefficients are real.

Introducing the series expansion (1) in Maxwell's equations, the following equations can be deduced for the field in a ν-order approach:

$$\vec{\nabla}_t \times \vec{E}_{t,\nu} = -jB_{z,\nu-1}\vec{u}_z \quad (2a)$$

$$\vec{\nabla}_t \times \vec{H}_{t,\nu} = jD_{z,\nu-1}\vec{u}_z \quad (2b)$$

$$\vec{\nabla}_t E_{z,\nu} = -j \sum_{\substack{k,m \\ (k+m=\nu)}} \beta_k \vec{E}_{t,m} - j\vec{u}_z \times \vec{B}_{t,\nu-1} \quad (2c)$$

$$\vec{\nabla}_t H_{z,\nu} = -j \sum_{\substack{k,m \\ (k+m=\nu)}} \beta_k \vec{H}_{t,m} + j\vec{u}_z \times \vec{D}_{t,\nu-1} \quad (2d)$$

$$\vec{\nabla}_t \cdot \vec{D}_{t,\nu} = j \sum_{\substack{k,m \\ (k+m=\nu)}} \beta_k D_{z,m} \quad (2e)$$

$$\vec{\nabla}_t \cdot \vec{B}_{t,\nu} = j \sum_{\substack{k,m \\ (k+m=\nu)}} \beta_k B_{z,m} \quad (2f)$$

where

$$\vec{D}_\nu = \sum_{\substack{k,\nu \\ (k+m=\nu)}} \bar{\bar{\epsilon}}_k \cdot \vec{E}_m \quad (3a)$$

$$\vec{B}_\nu = \sum_{\substack{k,m \\ (k+m=\nu)}} \bar{\bar{\mu}}_k \cdot \vec{H}_m \quad (3b)$$

where the subscript t indicates, as usual, the projection in the transverse $x-y$ plane, the subscript z the z-component, and the subscripts ν, k, and m the order of the coefficients as in (1). From (3), the same properties indicated above for the complex field terms in the series expansion (1) hold also for the series coefficients of $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$.

The "static" zero-order ($\nu = 0$) solution of (2) corresponds to transverse electrostatic and magnetostatic fields ($E_{z,0} = 0$, $H_{z,0} = 0$), which can be obtained from the following transverse electric and magnetic (ϕ, Ψ) potentials:

$$\vec{E}_{t,0} = -\vec{\nabla}_t \phi \qquad \vec{\nabla}_t \cdot \bar{\bar{\epsilon}}_{t,0} \cdot \vec{\nabla}_t \phi = 0 \quad (4a)$$

$$\vec{H}_{t,0} = -\vec{\nabla}_t \Psi \qquad \vec{\nabla}_t \cdot \bar{\bar{\mu}}^{-1} \cdot \vec{\nabla}_t \Psi = 0. \quad (4b)$$

If $\vec{u}_z$ coincides with a principal axis of $\bar{\bar{\epsilon}}_0$ and $\bar{\bar{\mu}}_0$, it is also true that $D_{z,0} = B_{z,0} = 0$. If $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ are real and $\vec{u}_z$ coincides with a principal axis of $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ in a certain range of frequencies, all the transverse components of the complex field vectors will be even (and real) functions of frequency and all the longitudinal components will be odd (and imaginary) functions of frequency. In this case, the static approach for the transverse fields holds up to a second order in ω.

B. Quasi-Static Approach: TEM Modes

The solutions to (2) in the zero-order approach defines the field in the widely used quasi-static approach. For the propagation factor, we take $\nu = 1$ and integrate (2c) between A and B (see Fig. 1), and (2d) along a path enclosing the ith conductor, obtaining, in a similar way as in [1]

$$\beta_1 V_i = L_{i,j} I_j \quad (5a)$$

$$\beta_1 I_i = C_{i,j} V_j \quad (5b)$$

where V_i is the electrostatic potential at the ith conductor, referred to the $N+1$, I_i the magnetostatic intensity on the ith conductor, and $L_{i,j}$ and $C_{i,j}$ the inductance and capacitance matrices, respectively. All these quantities referred to the zero-order field obtained from (2) with $\nu = 0$, or from (3).

From (5), we obtain now

$$\beta_{1,n}^2 \bar{V}_n = \bar{\bar{L}} \cdot \bar{\bar{C}} \cdot \bar{V}_n \quad (6a)$$

$$\beta_{1,n}^2 \bar{I}_n = \bar{\bar{C}} \cdot \bar{\bar{L}} \cdot \bar{I}_n \quad (6b)$$

where the subscript n refers to each of the quasi-static propagating modes.

We have not yet shown that simultaneous real and positive solutions to the eigenvalue problems (6) exist. Nevertheless, if the static conductivity of the medium is taken to be zero, the first terms in the series expansion of $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ (i.e., $\bar{\bar{\epsilon}}_0$ and $\bar{\bar{\mu}}_0$) must be real and symmetric, since all terms involving losses are functions of frequency [3]. Then $\bar{\bar{L}}$ and $\bar{\bar{C}}$ are symmetric (see Appendix). On the other hand, as $\bar{\bar{L}}$ and $\bar{\bar{C}}$ have to be positive matrices, $\bar{\bar{L}}$ and $\bar{\bar{C}}$ must be diagonalizable with the same set of eigenvalues, being themselves real and positive [5]. Then, there are N different quasi-static propagating modes, with propagation factors $\omega\beta_{1,n}$ ($n = 1, 2, \ldots, N$). The quasi-static voltages and intensities of such modes are given by the eigenvectors $\bar{V}_n$ and $\bar{I}_n$ in (6).

The coupling between the electric and magnetic quasi-static fields can be expressed through an impedance matrix $\bar{\bar{Z}}$. Only when $\bar{\bar{L}}$ and $\bar{\bar{C}}$ commute can each mode be characterized by a scalar impedance Z_n [2]. In the most general case, both eigenvector problems (5a) and (5b) do not have the same solutions, and $\bar{V}_n = \bar{\bar{Z}}\bar{I}_n$, where $\bar{\bar{Z}}$ is a symmetric matrix characterized by [2]

$$\bar{\bar{L}} = \bar{\bar{Z}} \cdot \bar{\bar{C}} \cdot \bar{\bar{Z}}. \quad (7)$$

C. Transmission Lines in Nonmagnetic Media

In a nonmagnetic medium, $\bar{\bar{L}} = \bar{\bar{L}}_v$, where $\bar{\bar{L}}_v$ is the inductance matrix for the structure "in vacuum," that is, when the material medium is removed. Furthermore, $\bar{\bar{L}}_v \cdot \bar{\bar{C}}_v = 1/c^2$. Taking this into account, (6) and (7) can be written:

$$c^2 \cdot \beta_{1,n}^2 \bar{V}_n = \bar{\bar{C}}_v^{-1} \cdot \bar{\bar{C}} \cdot \bar{V}_n \quad (8a)$$

$$c^2 \beta_{1,n}^2 \bar{I}_n = \bar{\bar{C}} \cdot \bar{\bar{C}}_v^{-1} \cdot \bar{I}_n \qquad (n = 1, 2, \cdots, N) \quad (8b)$$

$$\bar{\bar{C}}_v^{-1} = c^2 \bar{\bar{Z}} \cdot \bar{\bar{C}} \cdot \bar{\bar{Z}}. \quad (9)$$

Equations (8) and (9) obviously generalize well-known equations from elementary theory. Equations (8) and (9) also show that the knowledge of the capacitance matrix $\bar{\bar{C}}$ and $\bar{\bar{C}}_v$ is sufficient to know the circuital properties of general transmission lines in nonmagnetic media at the usual frequencies. Propagating modes with a scalar impedance occur when common eigenvectors for $\bar{\bar{C}}$ and $\bar{\bar{C}}_v$ exist. In the most general case, an impedance matrix (9) must be defined to relate $\bar{V}_n$ and $\bar{I}_n$.

III. Application to MIC Lines

A. General Method of Analysis

If we restrict ourselves to a quasi-static analysis, the propagating modes can be analyzed using (8) and (9). So, our principal task is to determine the $\bar{\bar{C}}$ and $\bar{\bar{C}}_v$ matrices of the structure.

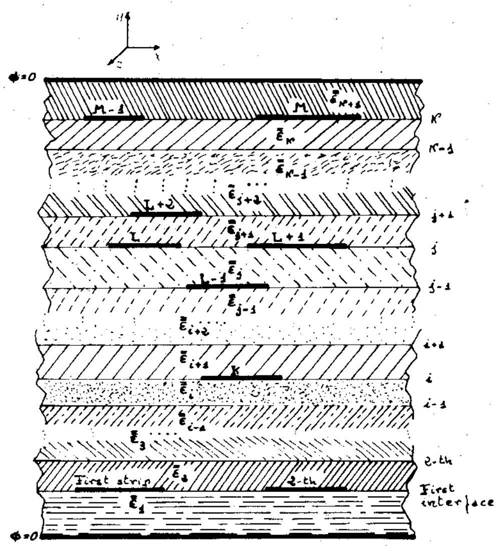

Fig. 2. General open planar dielectric transmission line of $M+1$ conductors and $N+1$ layers.

Recently, the authors have developed a general method for determining the static Green's function for any planar open structure with arbitrary anisotropic layers [6] (Fig. 2). This algorithm permits us to construct a general method to evaluate $\bar{\bar{C}}$.

First we define the quantities

$$U_{K,L}=\frac{1}{4\pi}\sum_{i=1}^{N'}\sum_{j=1}^{N'}\int_{-\infty}^{\infty}\tilde{\rho}_i^{+}(\alpha)\tilde{\bar{\bar{G}}}_{i,j}\tilde{\rho}_j(\alpha)\,\delta\alpha \quad (10)$$

where the superscript + indicates a complex conjugate, and where N' is the number of interfaces having conducting strips, $\tilde{\rho}_i(\beta)$ and $\tilde{\rho}_j(\beta)$ are the Fourier transforms of the true charge densities at ith and jth interfaces, when only the Kth and the Lth strips have got a neat charge Q, and $\tilde{\bar{\bar{G}}}_{i,j}(\beta)$ are the transformed Green's functions that can be calculated using the algorithm in [6]. These expressions are variational, and provide an accurate method for evaluating the $U_{K,L}$ when appropriate trial functions for the $\rho_i(x)$ are used.

The $U_{K,L}$ are actually electrostatic energies of some particular charge configurations of the structure. Therefore, they are related with the elements of the inverse matrix $\bar{\bar{C}}^{-1}$ by means of

$$U_{K,K}=\frac{1}{2}C_{K,K}^{-1}Q^2 \quad (11a)$$

$$U_{K,L}=\left(\frac{1}{2}C_{K,K}^{-1}+\frac{1}{2}C_{L,L}^{-1}+C_{K,L}^{-1}\right)Q^2. \quad (11b)$$

These relations permit us to calculate the $\bar{\bar{C}}^{-1}$ matrix for any open structure. Now using (8) and (9), the quasi-static parameters can be easily calculated.

An interesting consequence of expressions (2) and (3) is that in the zero-order approach, the fields are determined only by the projections of the constitutive tensors $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ over the transverse $x-y$ plane. This fact permits us to evaluate the quasi-static characteristic parameters of the line for an arbitrary orientation of the principal axes, taking into account only the transverse tensors $\bar{\bar{\epsilon}}_t$ and $\bar{\bar{\mu}}_t$, as long as the expansion (1) is valid. In particular, the characteristic parameters of microstrip-like structures on monoaxic substrates with its optical axis lying parallel to the ground plane can be calculated from the diagonal tensor $\bar{\bar{\epsilon}}_t$. This fact could be useful to vary the line characteristics of structures (such as shown in Fig. 3(a)) by varying the orientation of the strip on the substrate.

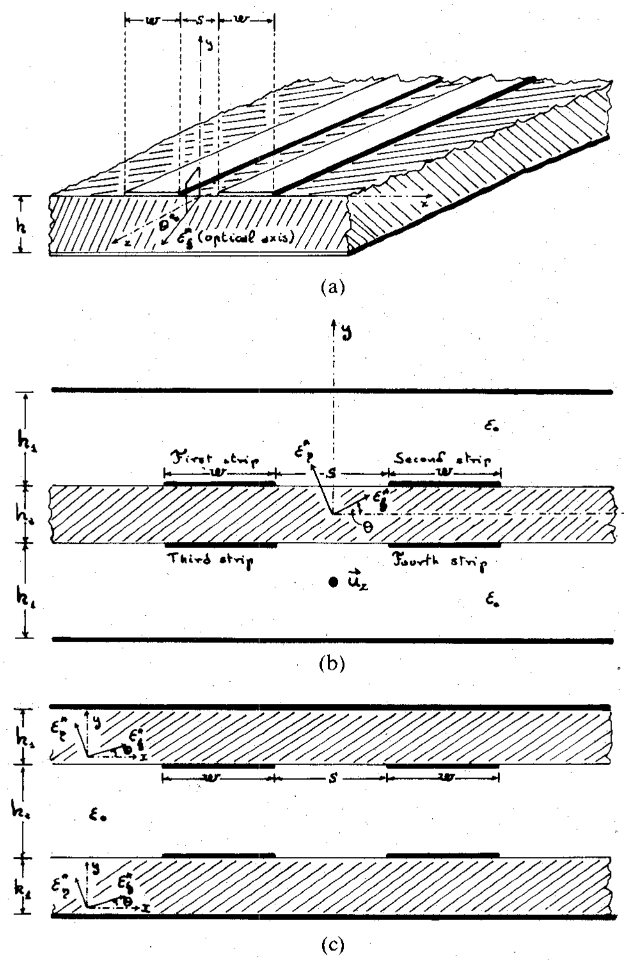

Fig. 3. (a) Coupled pairs of microstrip lines on monoaxic dielectric substrate with its optical axis lying parallel to the ground plane at an arbitrary orientation. (b) Broadside edge-coupled microstrips with an inverted substrate of tilted anisotropic dielectric. (c) Broadside edge-coupled microstrips with noninverted substrates of tilted anisotropic dielectric.

B. Example: The Broadside Edge-Coupled Microstrip Lines

We have chosen this structure because it presents all the most important features discussed in Section I. The broadside edge-coupled microstrip line with an inverted dielectric substrate (Fig. 3) is a nonsymmetrical structure with respect to reflection about the central ($y=0$) plane, if the principal axes of $\bar{\bar{\epsilon}}_t$ are tilted in the transverse $x-y$ plane. Therefore, the capacitance matrices $\bar{\bar{C}}$ and $\bar{\bar{C}}_v$ will not have the same set of eigenvectors and will not commute. So, the usual even–even, even–odd, etc., modes of propagation do not exist, nor can a scalar impedance be defined for each propagating mode.

Nevertheless, the structure remains the same after inversion with respect to the origin of coordinates ($x=0,\ y=0$)

(see Fig. 3). So, using a notation similar to that used in [7], the four $c-c$, $c-\pi$, $\pi-c$, and $\pi-\pi$ propagating modes can be defined. If the quasi-static voltage eigenvector of each mode is defined as the vector whose components are the voltages at the first, second, etc., strips, then

$$\overline{V}_{c-c} = (v_1, v_2, v_2, v_1) \quad (12a)$$

$$\overline{V}_{c-\pi} = (v_3, v_4, -v_4, -v_3) \quad (12b)$$

$$\overline{V}_{\pi-c} = (v_5, -v_6, v_6, -v_5) \quad (12c)$$

$$\overline{V}_{\pi-\pi} = (v_7, -v_8, -v_8, v_7) \qquad (v_i > 0) \quad (12d)$$

and the four propagating modes can be defined by their four quasi-static phase velocities $v_{F,n} = 1/\beta_{s,n}$ and by the four quantities:

$$R_{c-c} = \log_{10}\left(\frac{v_1}{v_2}\right) \quad (13a)$$

$$R_{c-\pi} = \log_{10}\left(\frac{v_3}{v_4}\right) \quad (13b)$$

$$R_{\pi-c} = \log_{10}\left(\frac{v_5}{v_6}\right) \quad (13c)$$

$$R_{\pi-\pi} = \log_{10}\left(\frac{v_7}{v_8}\right). \quad (13d)$$

In the particular case in which the projection $\bar{\bar{\epsilon}}_t$ is diagonal, $R_{c-c} = R_{c-\pi} = R_{\pi-c} = R_{\pi-\pi} = 0$, and the modes defined above coincide with the usual even–even, even–odd, odd–even, and odd–odd modes.

C. *Numerical Results*

To compute the characteristic parameters of the two structures analyzed, we have used the method proposed in this paper with similar trial functions as in [6] for the charge density on the strips, and the Rayleigh–Ritz procedure to minimize (10). The results have been compared with previous ones [8]–[10] for the limiting case when $\bar{\bar{\epsilon}}_t$ is diagonal, or the medium is isotropic, and are in good agreement.

Fig. 4 shows the variation of normalized phase velocities and impedances for coupled microstrips on a sapphire substrate, cut with its optical axis parallel to the ground plane, as a function of the angle θ between the optical axis and the strip orientation. The behavior of these parameters is similar to that found in [8], [9] for a substrate tilted around the direction of propagation.

Figs. 5 and 6 show the variation of the mode phase velocities and the R factors, respectively, for the broadside edge-coupled microstrip lines with inverted dielectric substrates of boron nitride, tilted around the direction of propagation, as a function of the tilting angle. It can be seen that the R parameters soon deviate from their nontilted values ($R = 0$), and that the deviation is strong for certain modes. These results show that the nontilted even–even,..., modes are inappropriate to describe electromagnetic propagation in this kind of structure with tilted substrates.

Finally, in Fig. 7, the variation of phase velocities for propagating even–even, even–odd,..., etc., modes in the

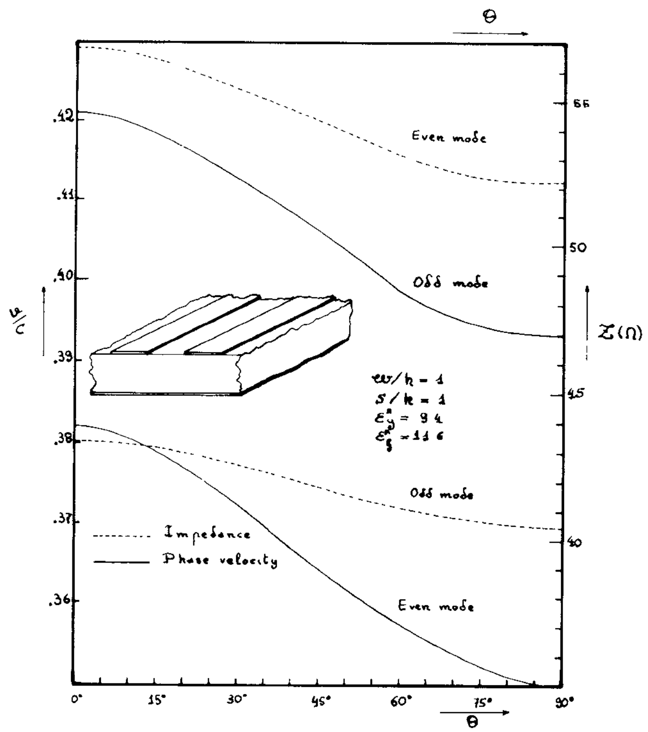

Fig. 4. Variation of the normalized mode phase velocities and impedance for each mode in the coupled pairs of microstrips on monoaxic dielectric with its optical axis parallel to the ground plane, as functions of the angle between the optical axis and the strip orientation (substrate sapphire: $\epsilon_\xi^* = 11.6$, $\epsilon_\eta^* = 9.4$).

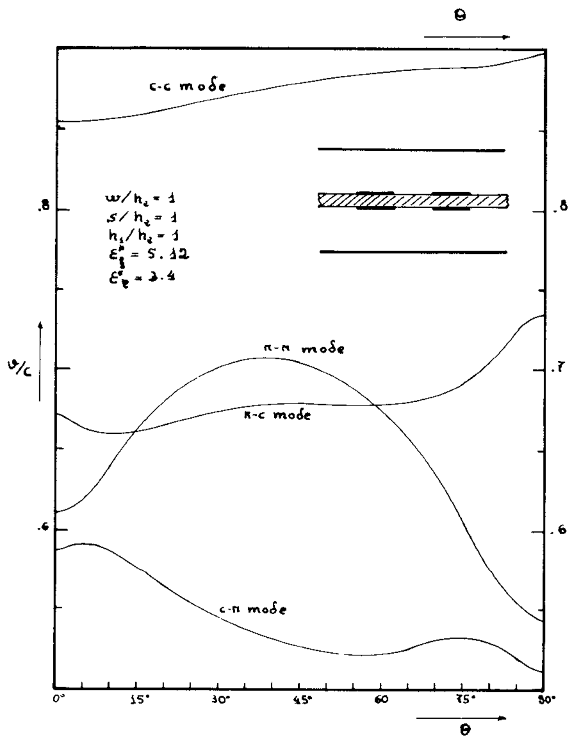

Fig. 5. Variation of the R-parameters in the broadside edge-coupled microstrips with an inverted substrate of boron nitride, as functions of the tilting angle of the substrate ($\epsilon_\xi^* = 5.12$, $\epsilon_\eta^* = 3.4$).

broadside edge-coupled microstrips with noninverted substrates of boron nitride is plotted as a function of the tilting angle. In this case, the structure behaves as a symmetrical coupling of two pairs of coupled microstrips (a

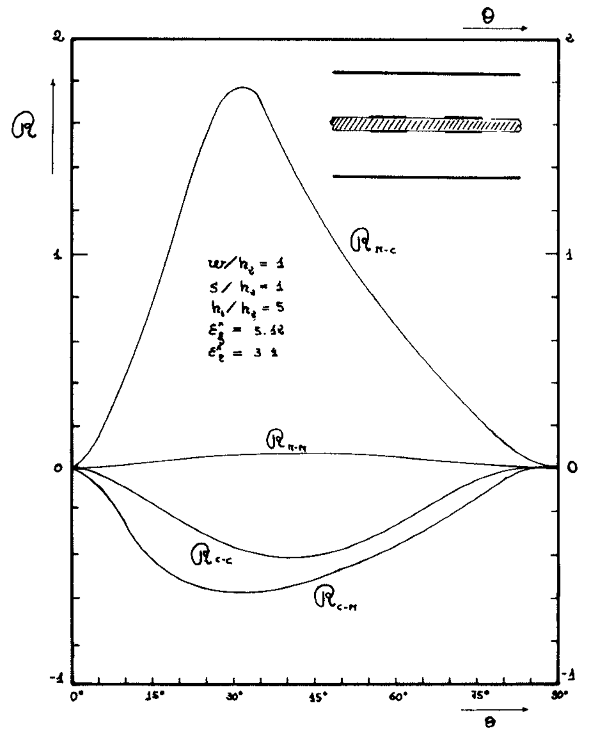

Fig. 6. Variation of the normalized mode phase velocities in the broadside edge-coupled microstrips with an inverted substrate of boron nitride, as functions of the tilting angle ($\epsilon_{\xi}^{*}=5.12$, $\epsilon_{\eta}^{*}=3.4$).

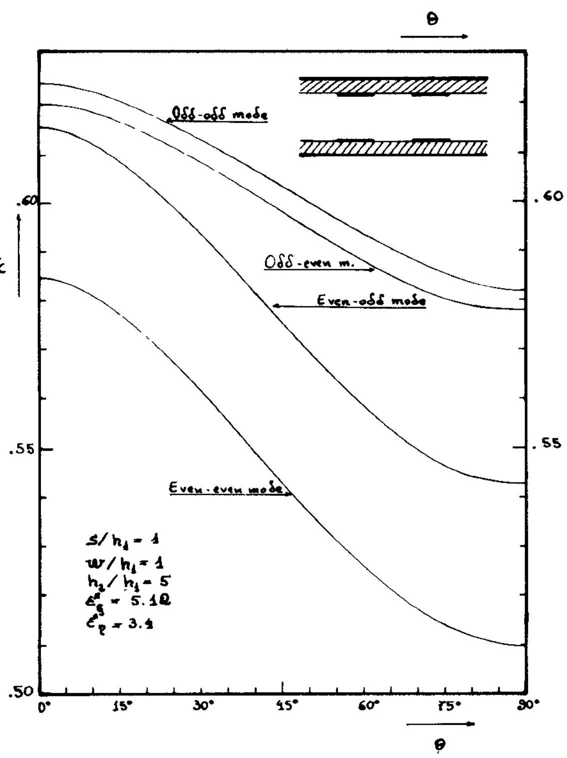

Fig. 7. Variation of the normalized mode phase velocities in broadside edge-coupled microstrips with noninverted substrates, as functions of the tilting angle ($\epsilon_{\xi}^{*}=5.12$, $\epsilon_{\eta}^{*}=3.4$).

structure which propagates even and odd modes, even for tilted substrates, as has been verified), and the usual even–even,..., modes can propagate, as has been checked. It has also been verified that the mode characteristics do not vary if the tilting angles of the two substrates are the same, or the same but of opposite sign. These results are expected from the form of Green's functions in [6].

IV. Conclusions

An analytical justification of the quasi-static approach for multiconductor transmission lines in inhomogeneous anisotropic media is made by using the series expansion of the field in powers of frequency.

It is shown, in a closed form, that it is even possible to decompose the quasi-static field into a sum of propagating modes with a scalar propagation factor, and a method to find out such modes is developed, provided the static inductance and capacitance matrices of the line are known.

Most MIC lines (except for nonreciprocal devices) are impressed on nonmagnetic substrates. In this case, the matrices $\bar{\bar{C}}$ and $\bar{\bar{C}}_v^{-1}/c^2$ play the same role that $\bar{\bar{C}}$ and $\bar{\bar{L}}$ play in the general case. A variational method to compute $\bar{\bar{C}}$ and $\bar{\bar{C}}_v$ (and so, to characterize the quasi-static propagating modes) for any open planar dielectric line is developed and applied to some known MIC lines when they are impressed on tilted dielectric substrates. From the general analysis, it follows that only the projection on the transverse plane of the dielectric tensor (or permeability tensor in the magnetic case), must be taken into account to compute the circuital characteristics of the line in a quasi-static approach. When this projection is not a diagonal tensor, it causes broken symmetries. In some analyzed structures, propagating modes seem to be very sensitive to this fact.

Appendix

To show the symmetry of the matrix $\bar{\bar{C}}$, a similar procedure to the one followed in [4] can be used. We first consider the integral of electrostatic energy U_e

$$U_e=\frac{1}{2}\iiint \vec{\nabla}\phi\cdot\bar{\bar{\epsilon}}_0\cdot\vec{\nabla}\phi\,dv \tag{A1}$$

extended to all the space between conductors for a portion Δz of the line. The variation of U_e can be written

$$\delta U_e=\frac{1}{2}\iiint \vec{\nabla}\,\delta\phi\cdot\bar{\bar{\epsilon}}_0\cdot\vec{\nabla}\phi\,dv+\frac{1}{2}\iiint \vec{\nabla}\phi\cdot\bar{\bar{\epsilon}}_0\cdot\vec{\nabla}\,\delta\phi\,dv. \tag{A2}$$

If the varied field is to be a solution of the electrostatic problem as well, δU_e can be written (using Green's theorem)

$$\delta U_e=\frac{1}{2}\sum_i V_i\oint\!\!\oint\delta\left(\bar{\bar{\epsilon}}_0\cdot\vec{\nabla}\phi\right)\cdot d\vec{s}+\frac{1}{2}\sum_i \delta V_i\oint\!\!\oint\left(\bar{\bar{\epsilon}}_0\cdot\vec{\nabla}\phi\right)\cdot d\vec{s} \tag{A3}$$

where the integrals are extended over the entire surface of conductors. Now, taking into account the symmetry of $\bar{\bar{\epsilon}}_0$, the two terms on the right-hand side of (A2) (and (A3)), are equivalent, and δU_e can be written in the two equivalent forms

$$\delta U_e=\sum_i V_i\,\delta Q_i \tag{A4a}$$

$$\delta U_e=\sum_i Q_i\,\delta V_i. \tag{A4b}$$

Therefore, we can write $\partial U_e/\partial Q_i = V_i$ and $\partial U_e/\partial V_i = Q_i$. Taking this into account, we can write

$$\frac{\partial^2 U_e}{\partial V_i \partial V_j} = \frac{\partial Q_i}{\partial V_j} = C_{i,j}\Delta z \tag{A5}$$

and changing the order of derivation: $C_{ij} = C_{ji}$.

To show the symmetry of $\bar{\bar{L}}$, a dual procedure can be chosen. We first consider the N loops composed by the ith and the $N+1$ conductors. The integral of magnetostatic energy is defined now in terms of the potential vector as

$$U_m = \frac{1}{2}\iiint (\vec{\nabla}\times\vec{A})\cdot\bar{\bar{\mu}}_0^{-1}\cdot(\vec{\nabla}\times\vec{A})\, dv. \tag{A6}$$

Taking into account the symmetry of $\bar{\bar{\mu}}_0$, it can be shown that the variation δU_m can be written either in the form (invariance of the structure along z is considered)

$$\delta U_m = \sum_i \Phi_i \delta I_i \tag{A7}$$

or in the form

$$\delta U_m = \sum_i I_i \delta\Phi_i \tag{A8}$$

where I_i is the stationary current flowing along the ith loop, and Φ_i is the magnetic flux through this loop

$$\Phi_i = \iint \vec{B}\cdot d\vec{s} = \oint \vec{A}\cdot d\vec{l} \qquad \text{(through/along } i\text{th loop)} \tag{A9}$$

and the line integral can be chosen along a closed current line, that is, along z.

From (A7) and (A8), the symmetry of $\bar{\bar{L}}$ can be established in the same way that was followed for $\bar{\bar{C}}$.

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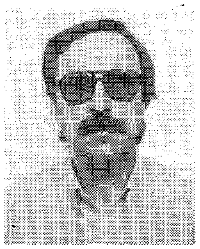

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